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Method of as-cast crack prediction within solidified layer *

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Abstract Based on the numerical simulation of solidification of castings, a thermal stress formula and a thermal crack initiation criterion are proposed. Using these formulas, cast steel wheels with a diameter of 800 mm and aluminum alloy electromagnetic casting (EMC) slabs with a size of 1300 mm × 480 mm are employed to testify the positions of cracks through conventional thermal elastic-plastic analyses and low magnifying structure observations. The results show that the numerical prediction of cracks agrees with the measured results, and the cracks do not necessarily occur on the defects such as shrinkage holes (wheel) and porosity (EMC slab). It is also found that surface temperature control is an effective means to avoid the crack formation.

Keywords: **solidification, numerical simulation, crack initiation, crack prediction.**

The as-cast crack is a kind of the serious defects, which is harmful to the quality of castings. Researchers have investigated it fruitfully by numerical simulations and quantitative analyses of stress fields with the thermal elastic-plastic method during solidification and resolved some practical problems[1~6]. However, the increment method is restricted to predicting some complicated crack problems, because some untraceable dynamic factors are ignored when establishing constitutive equations and the temperature field that served as the thermal load varies with time. Actually, the numerical simulation of dynamic stress fields is so complicated and the convergence criterion is so strict that it can hardly be grasped, and thus the applications of these methods are limited.

In this paper, a criterion of the as-cast crack initiation position and an analytical formula of thermal stresses within a solidified layer are proposed and the crack predictions for a cast steel wheel of ϕ800 mm and an EMC slab are conducted. The predicted results are evaluated with conventional thermal elastic-plastic analyses and low magnifying structure observations. It is found that the predicted results are in good agreement with the measured ones.

1 Constitutive equations

1.1 Temperature distribution

A lot of simulation results of the solidification process of casting have indicated that the temperature distribution of the solidified layer could be approximately considered as linear[1], so the temperature at point x apart from the surface can be given by

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$$T = T_0 + (T_S - T_0)\frac{x}{S}, \tag{1}$$

where, T_0 is the surface temperature of a casting, S the thickness of the solidified layer, T_S the solidus temperature.

1.2 Stress distribution

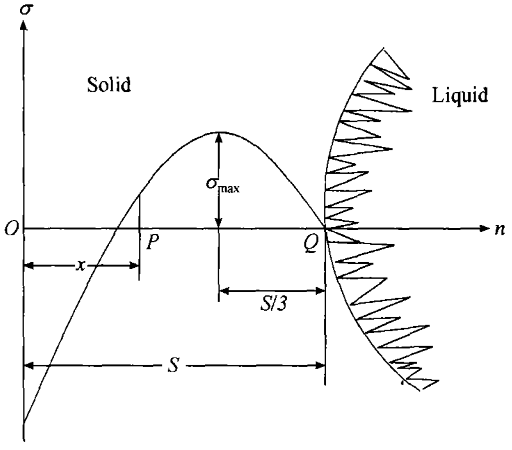

Fig. 1 Schematic view of thermal stress formation.

The thermal stress can be derived from the dynamic solidification zone, as shown in Fig. 1. The following assumptions are made for the present model: (i) the cross section remains a plane, (ii) the rotation angle of cross section is zero, (iii) the product of the modules of elasticity and the linear coefficient of expansion is constant, and (iv) the hydrostatic pressure of melt is ignored.

Let the surface temperature be T_0, and the solidified layer thickness be S at time t. The surface temperature is $T_0 + \mathrm{d}T_0$, and the layer thickness is $S + \mathrm{d}S$ at time $t + \mathrm{d}t$. Then the strain increment $\mathrm{d}\varepsilon_Q$ at the point Q of the solidifying front and the strain and stress increment at point P in this layer may be written as

$$\mathrm{d}\varepsilon_P = \frac{x}{S}\mathrm{d}\varepsilon_Q, \tag{2}$$

$$\mathrm{d}\sigma_P = E(\mathrm{d}\varepsilon_P - \alpha\mathrm{d}T) = E\left(\frac{x}{S}\mathrm{d}\varepsilon_Q - \alpha\mathrm{d}T\right). \tag{3}$$

Since

$$\mathrm{d}T = \left(1 - \frac{x}{S}\right)\mathrm{d}T_0 - \frac{x}{S^2}(T_S - \mathrm{T}_0)\mathrm{d}S, \tag{4}$$

we may write

$$\mathrm{d}\sigma_P = E\left\{\frac{x}{S}\mathrm{d}\varepsilon_Q - \alpha\left[\left(1 - \frac{x}{S}\right)\mathrm{d}T_0 - \frac{x}{S^2}(T_S - T_0)\mathrm{d}S\right]\right\}. \tag{5}$$

Therefore, the bending moment of $\mathrm{d}\sigma_P$ about the point 0 is given by

$$\mathrm{d}M_0 = \int_0^S \mathrm{d}\sigma_P \cdot \mathrm{d}x \cdot x = E\left\{\frac{S^2}{3}\mathrm{d}\varepsilon_Q - \alpha\left[\frac{S^2}{6}\mathrm{d}T_0 - \frac{S}{3}(T_S - T_0)\mathrm{d}S\right]\right\}. \tag{6}$$

Since $\mathrm{d}M_0 = 0$, according to assumption (2),

$$\mathrm{d}\varepsilon_Q = \alpha\left(\frac{1}{2}\mathrm{d}T_0 - \frac{T_S - T_0}{S}\mathrm{d}S\right). \tag{7}$$

Then eq. (5) becomes

$$d\sigma_P = E\alpha\left(\frac{3x}{2S} - 1\right) dT_0. \tag{8}$$

Since $\sigma_P = 0$, at the solidifying front

$$\sigma_P = -\int_X^S d\sigma_P = -\int_X^S E\alpha\left(\frac{3x}{2S} - 1\right)\frac{dT_0}{dS} dS. \tag{9}$$

From Eq. (4) we may see that, if boundary temperature increases by dT_0, then the temperature increment caused by the increment dS of solidified layer thickness is $-\frac{x}{S^2}(T_S - T_0)dS$ at the point P. Thus

$$\frac{dT_0}{dS} = -\frac{x}{S^2}(T_S - T_0). \tag{10}$$

In this case, Eq. (9) may be written as

$$\sigma_P = -\frac{E\alpha}{4}(T_S - T_0)\left(1 - \frac{3x}{S}\right)\left(1 - \frac{x}{S}\right). \tag{11}$$

Eq. (11) is just the stress formula within the solidified layer.

2 Criterion of thermal crack initiation

Formula (11) indicates that the stress is a quadratic function of the corresponding coordinates. It also shows that the zone near the solidifying front is under the tensile condition and the surface is under compression during the advancing of the solidifying front, as can be seen from Fig. 1. Thus, when

$$x = \frac{2S}{3}, \tag{12}$$

the stress is the highest, and its value is

$$\sigma_{\max} = \frac{E\alpha}{12}(T_S - T_0). \tag{13}$$

Therefore, during the advancing of solidifying front, the criterion of crack initiation is that if

$$\sigma_{\max} = \sigma_b, \tag{14}$$

then the crack is initiated at the point of two-thirds of thickness of solidified layer. It should be noticed that the tensile strength σ_b is a function of the temperature at $x = 2S/3$. The above-mentioned results show that with the advancing of solidifying front, the stress reaches its maximum value at the point of one-thirds of distance from the solidifying front, and the maximum value depends on the surface temperature T_0 of the casting.

3 Crack judgment based on numerical simulation of solidification

Figure 2 shows the numerical simulation result of solidification of an actual ϕ800 cast steel wheel which was worked out by the cold loop technology. The final solidification zone of the wheel (not including the riser) is located at the hub near the rib. In the light of the crack criterion formulas (14) and (12), a crack zone, the shadow zone shown in Fig. 2, should be located at the point of two-thirds of distance from the outer surface of the wheel to the final solidification zone. Fig. 3 (a) shows the actual crack section of the wheel, and the crack section agrees with the predicted zone in Fig. 2. Fig. 3 (b) shows the low magnifying structure of a section of the wheel (not fractured). It can be seen that the macro-segregation grooves formed though this section is not fractured. Due to the decrease of cooling rate near the hub, the zone "A" lies in the maximum stress zone for a longer time. So some micro-cracks occur in the zone "A". Consequently, the solute elements and impurity elements such as P and S converge on these micro-cracks which may expand constantly to finally form macro-segregation grooves, which become the crack sources. If the stress in this zone is larger than its tensile strength, then the casting fractures.

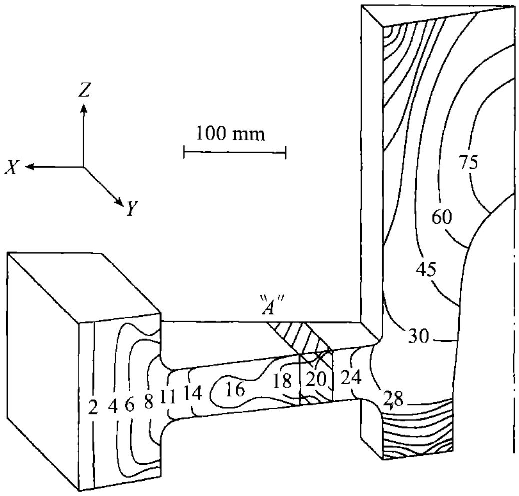

Fig. 2 The isochronal solidification profiles of the steel wheel ("A" indicates expected area of the cracking).

On the other hand, Fig.3 (b) shows that there is a crescent shrinkage hole at the junction of the rib and the rim, and the junction is just the corner where the stress concentration develops, but there is not any actual rupture behavior in this junction. This fact suggests that the cracks do not bear a direct relation to structure defects, and the crack initiation during solidification is determined by Formulas (14) and (12). If a crack source is exactly located at the defect zone, the defect will speed up the expansion of the crack according to the theory of fracture mechanics. If the defect zone is not a crack source, the defect zone will not rupture.

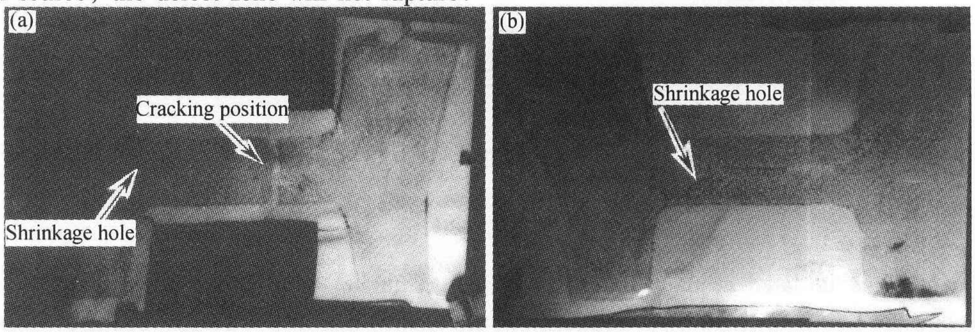

Fig. 3 Low magnifying structure in cross section of steel wheel. (a) Cross section with cracking; (b) cross section without cracking.

Formula (11) also indicates that the surface temperature is a key factor to avoid the crack formation. The statistical result in the spot shows that in an icy winter, the reject rate is up to 30% (usually 6%—8%); but in a torrid summer, the reject rate is near to zero. This fact shows that the sharp decrease of surface temperature results in the increase of the probability of crack formation.

4 Verification of thermal crack initiation criterion

4.1 Verification based on numerical simulation of stress field

The simulated results of temperature fields serve as input for the TEM-STRESS soft package as the thermal load, and its control equation is

$$[\boldsymbol{K}]_i \Delta\{\boldsymbol{q}\}_i = \Delta\{\boldsymbol{p}\}_i\{\boldsymbol{R}\}_{i-1}, \tag{15}$$

where $[\boldsymbol{K}]_i$, $\Delta\{\boldsymbol{q}\}_i$, $\Delta\{\boldsymbol{p}\}_i$ and $\{\boldsymbol{R}\}_i$ are the gross rigidity matrix of the i-th increment load, node displacement matrix, gross load matrix and initial disequilibrium mending term respectively.

The calculated result of the large cast steel wheel is shown in Fig. 4, which reveals a stress field at 60 min after the pouring. It shows that the maximum stress zone agrees with the predicted result in Sec. 2. All these show that the crack initiation can be predicted based on the numerical simulation of solidification.

4.2 Verification based on residual stress distribution of wheel

Figure 5 shows the measured distribution of the residual stresses of the ϕ800 mm wheel. The ribnear the hub has higher stresses, which gradually decrease along the radius from the hub to the rim. The higher stress zone agrees well with the crack initiation zone predicted in Fig. 2. This also shows that the crack initiation can be predicted based on the numerical simulation of solidification, and according to the predicted result, the actual technology can be improved to be free of crack initiation and expansion.

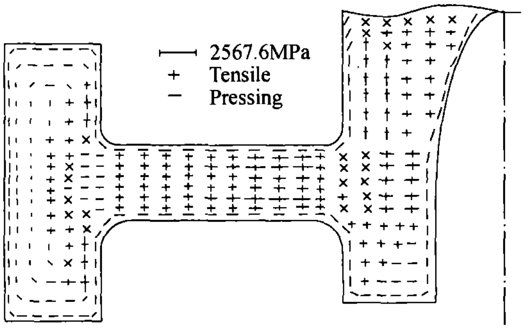

Fig. 4 Distribution of the thermal stress in a steel wheel.

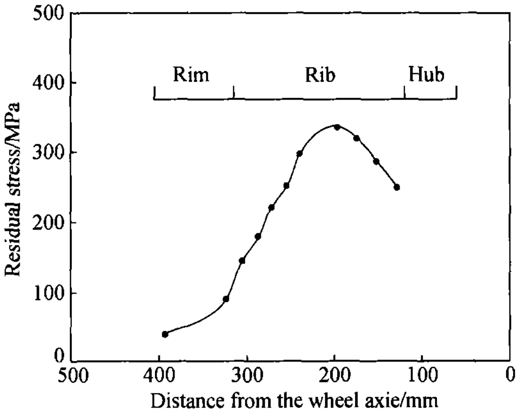

Fig. 5 Distribution of residual stress of the steel wheel.

4.3 Verification based on cross section observation of EMC slab

The cross section of an aluminum alloy electromagnetic casting (EMC) slab with a size of 1300 mm × 480 mm is shown in Figure 6.

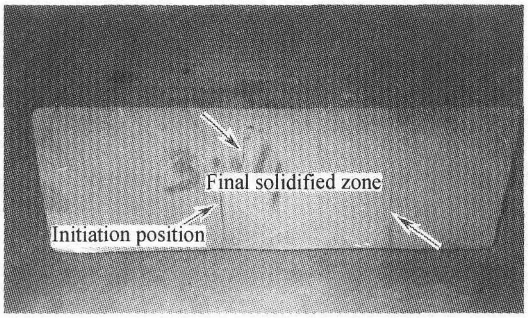

Fig. 6 Position of cracks in EMC slab.

Its final solidification zone is located at the centreline of the wider surface. According to Formulas (12) and (14), the cracks should be initiated at the point of two-thirds of thickness of solidified layer from the wider surface. Lots of cracks initiated at the corresponding zone (shown in Fig. 6) indicate that the crack initiation criterion is valid. It can also be found that the central porosity zone is not the crack initiation zone. During the final stage of solidification, if the crack initiation zone is under tensile stress, the micro-cracks will rapidly expand according to the theory of fracture mechanics. Consequently, the macro-cracks are formed.

5 Conclusions

(i) The as-cast thermal crack starts during the solidification, and it is located at the point of two-thirds of thickness of solidified layer from outside of a casting. The key factor to avoid as-cast cracks is to control the surface temperature of a casting during solidification.

(ii) The criterion of as-cast cracks is $\sigma_{\max} \geqslant \sigma_b$, where σ_b is the tensile strength of the highest stress zone at the corresponding temperature.

(iii) The shrinkage holes and porosities are not the necessary conditions to induce the as-cast cracks. The numerical simulation method of solidification is simple, and can be easily used to predict the as-cast cracks.

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